Logic and Mathematics

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Learning Outcomes

- Describe basic principles Deduction, Induction and Abduction
- Relate Logic and Mathematics to Philosophy of Science

Logic

- The study of relationship between thoughts
- We use logic daily
  - making mental connections between events
  - make decisions from beliefs
  - take actions based on decisions

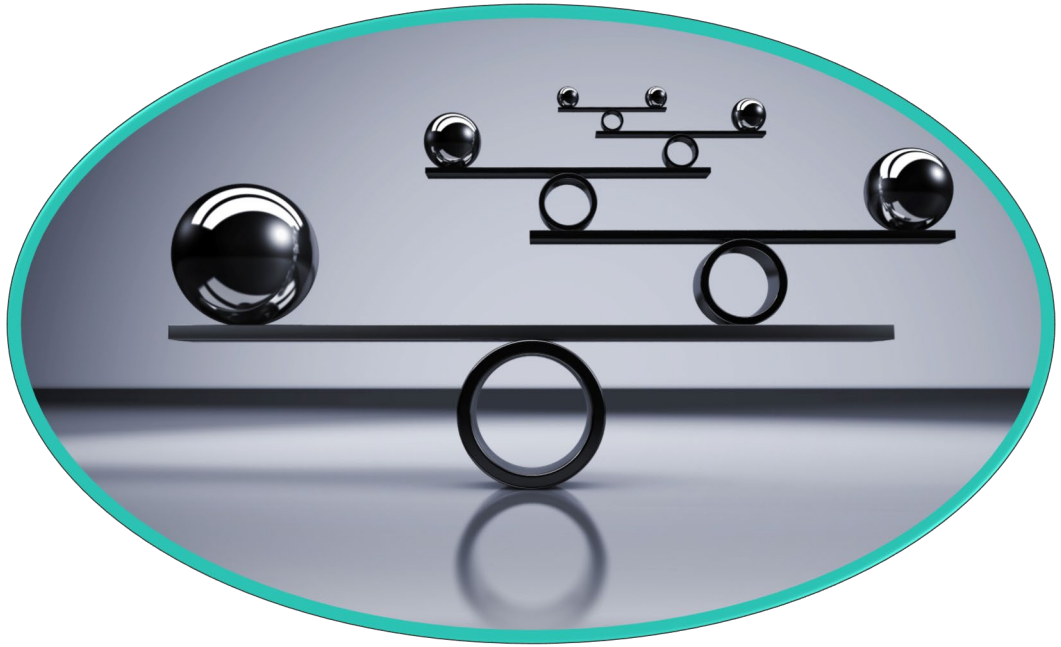
- Inference is a movement between thoughts
  - Premise 1: Layla is Marc's wife
  - Conclusion: Layla knows of Marc

- Inference may involve presuppositions or hidden premises
  - Premise 1: Layla is Marc's wife
  - Hidden Premise 2: Wives know of their husbands
  - Conclusion: Layla knows of Marc

- Premise
  - statement that has truth-value
  - ‘Please take a seat’ is not a premise
  - ‘Stephen is a surgeon’ is a premise

- Deduction
  - When the premises are true, the conclusion is necessarily true
    - P1: All surgeons are doctors
    - P2: Stephen is a surgeon
    - C: Therefore, Stephen is a doctor

- Deduction
  - Syllogism
  - modus ponens
  - modus tollens



- Syllogism has three components
  - Major premise
  - Minor premise
  - Conclusion
    - P1: All surgeons are doctors (major)
    - P2: Stephen is a surgeon (minor)
    - C: Therefore, Stephen is a doctor

- Modus ponens
  - contains conditional, ‘if-then’ statements

1. If P, then Q: If the virus infects the body, then the immune system responds.
2. P: The virus infects the body.

C. Therefore Q: Therefore, the immune system responds.

- Modus tollens
  - contains negating conditional statements
  1. If P, then Q: If the virus infects the body, then the immune system responds.
  2. Not Q: There is no immune response.
  C. Not P: Therefore, the virus did not infect the body.

- Validity and Soundness

Consider:

- P1: All surgeons are doctors
- P2: Stephen is a surgeon
- C: Therefore, Stephen is an Avenger

- Validity is about the structure, not content
- A valid argument
  - the truths of the premises structurally guarantee the truth of the conclusion
- Validity ignores the content of the premises

- Soundness

Consider:

- P1: All surgeons are cocaine addicts
- P2: Stephen is a surgeon
- C: Therefore, Stephen is a cocaine addict

- Soundness is about the structure and content
- A sound argument
    1. The structure is valid
    2. The content of the premises are true
- All sound arguments are valid, but not all valid arguments are sound

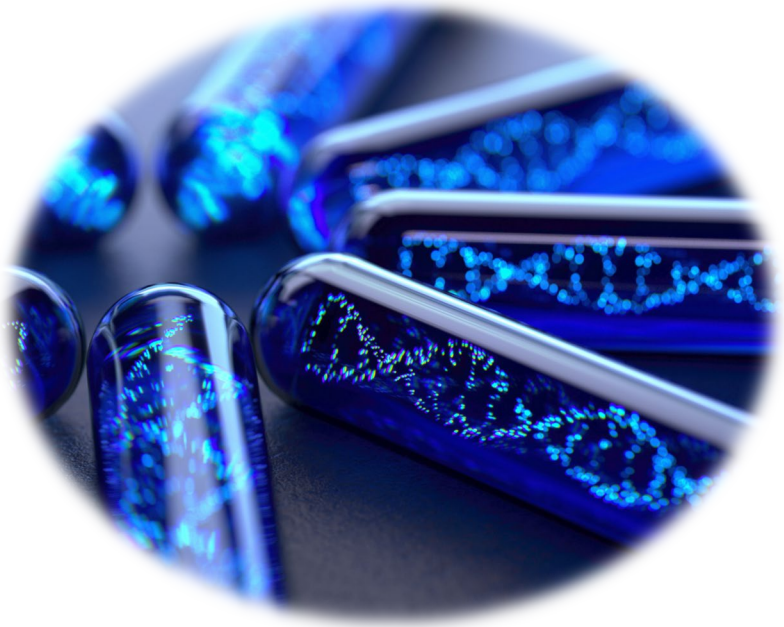
- Paradoxes
  - When reasoning leads to counterintuitive conclusions
- Logical paradoxes
  - ‘I am lying right now’ (Liar Paradox)

- Scientific paradoxes
  - Original Antigenic Sin
    - Immune system has memory
    - Stronger response against similar infection in the future
    - Weaker response against similar infection in the future

- Paradoxes highlight imperfections of reason
- Logical paradoxes sharpen our understanding of self-reference and its implications on truths.
- Scientific paradoxes challenge established theories

Induction

- When all the premises are true, the conclusion is probably true
- Inductive generalisation
  - Specific observations to general conclusion
  - Past observations to future observations



- Example:

1. This vaccine protects most clinical study participants against Virus X
2. I took this vaccine

C. Therefore, I am likely protected against Virus X

- Is it necessary that lab observations resemble phenomena in nature?
- Is it necessary that future observations resemble the past observations?

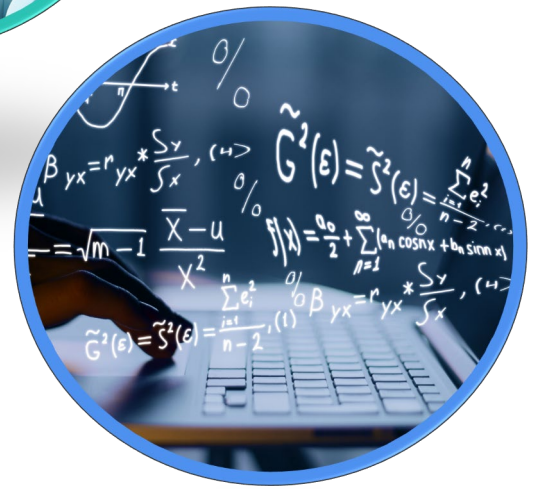
- Problem of Induction
- Empirical sciences cannot provide absolute truth
- Scientific theories are only expected to be true enough to
  - match our sense experience
  - make future predictions

- Abduction
  - from the same observations, scientists can infer more than one explanation
  - when this happens, scientists choose or infer to the best explanation
  - this inference to the best explanation is called Abduction

- P1: A patient presents with a runny nose, low fever and no other symptoms
- C: The patient has the common cold

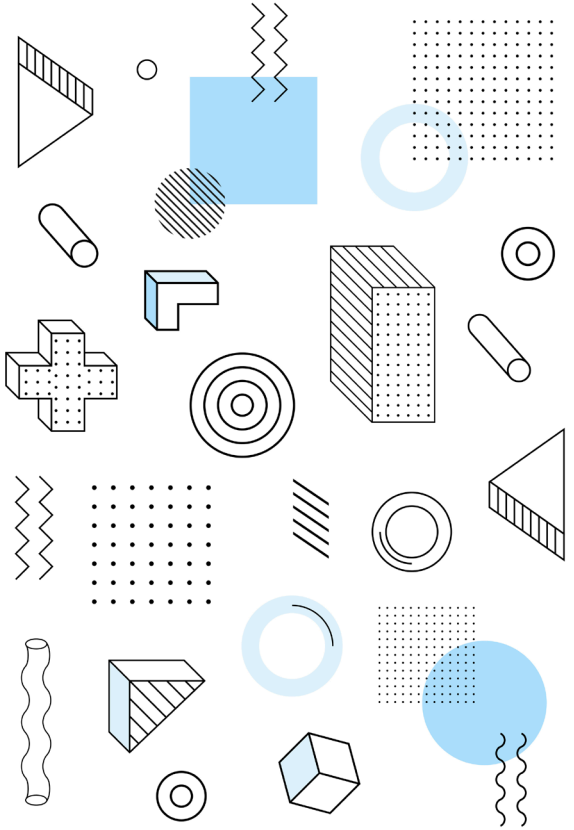
Formal Sciences

- Formal sciences study
  - abstract objects
  - formal systems



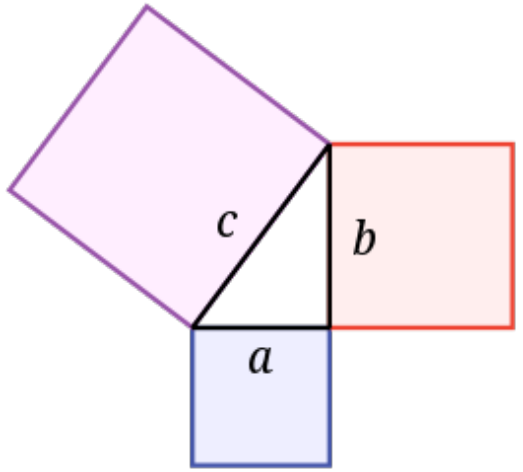
Mathematics

- Pure Mathematics
  - Numbers (rational, prime etc.)
  - Quantitative changes (calculus)
  - Numerical structure (algebra)
  - Shape (geometry)



- Applied Mathematics
  - Probability
  - Statistics
  - Blockchain

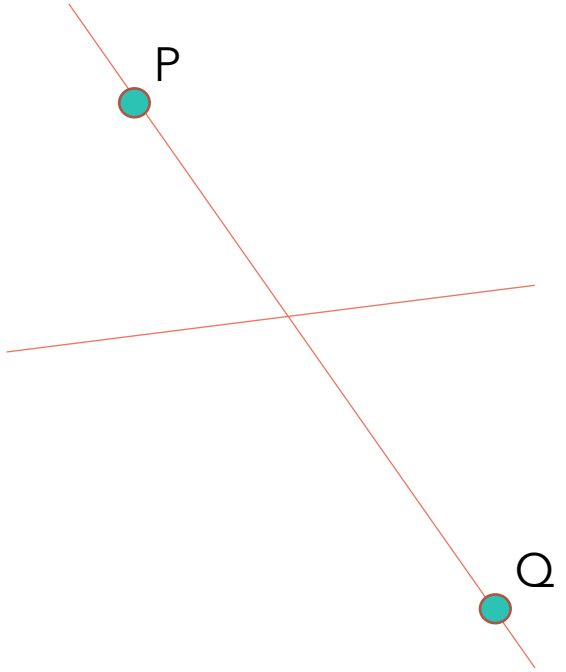
- Theorem
  - a mathematical statement that is accepted with proof
- Pythagorean theorem



- Gödel's Incompleteness Theorem
  - A complex formal system
  - which has a set of foundational statements and rules
  - can never be both complete and consistent at the same time.

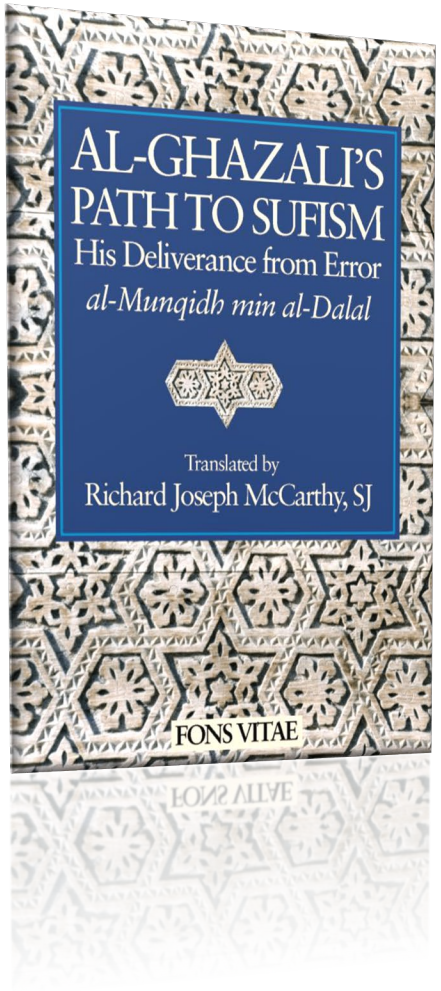
- Axiom and postulate
  - statement that is accepted as true without proof
  - used as the basis for mathematical reasoning

- E.g.:
  - For two points, there is exactly one straight line that can go through them
  - If you have two lines intersecting each other, they intersect at exactly one point



- Axioms outside Mathematics
  - Premises accepted without prerequisite argument
  - E.g.:
    - ‘You and I exist’
    - ‘not (P and not P)’

- Axioms vs empirical evidence
  - ‘10 is larger than 3’
  - even if you observe a rod turns into a serpent



- Why does Mathematics matter to Philosophy of Science?
  - Mathematics expresses patterns in nature
  - expressed in numerical language (quantitative strength)
  - Nature matters to science, so expressions of nature matters to Philosophy of Science

E.g., Mathematics informs our philosophy and experience:

1. Models for epidemiology
2. Bayesianism
3. Artificial Intelligence

4. Music

- C4 is 261.6256 Hz
- C5 is 523.2511 Hz